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ROBUST OPTIC FLOW ESTIMATION USING LEAST MEDIAN OF SQUARES

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ABSTRACT

A new approach to optic flow calculation, based on a highly robust statistical technique, is presented. In this algorithm, the optic flow problem is first formulated as a standard least squares problem. Then, its associated closest point problem is introduced and the transformation which takes this problem to a standard regression problem is provided. The Least Median of Squares technique is used to solve the resulting regression problem. Some experimental results for both synthetic and real image sequences are also presented.

1 INTRODUCTION

During the last two decades there has been an increasing interest in analysing image sequences, and, in particular, in recovering the optic flow field. Although many methods for estimating the flow field has been proposed, a practical robust solution to this problem remains a challenge. The current existing methods for the optic flow computation are mainly based on the following approaches:

- Correlation techniques
- Phase or energy techniques
- Differential techniques.

Differential techniques are often the prime candidates for real time applications due to computational ease and speed. The accuracy of any differential technique is mainly dependent on the accuracy of estimating derivatives of the image brightness function. The process of estimating derivatives of the brightness function for computing the optic flow is sensitive to noise. Thus, a robust approach to identify the corrupted estimates is vital.

The optic flow problem for sequential images can be formulated by assuming the temporal conservation of image brightness (its total derivative with respect to time is zero). This assumption results in the well-known optic flow constraint (OFC),

$$I_x u_x + I_y u_y + I_t = 0 \qquad (1.1)$$

where u_x and u_y are the translational velocities and I_x , I_y and I_t are partial derivatives of the image intensity function with respect to x, y and t. This equation shows that for every point in the image function, there exist two unknowns and one constraint, and, consequently, the number of solutions is unlimited (ill-posed problem).

In order to solve this problem, it is often assumed that the translational velocities are constant over a neighbourhood around the point where flow is estimated (the presented method can be easily extended to solve for other models of motion, e.g. affine). As a result, the component of image velocities can be estimated for a finite region by solving a set of over-determined linear equations as follows:

$$\mathbf{Ax} = \mathbf{b} \qquad (2.1)$$

where **A** is a two column matrix whose columns contain the intensity derivatives with respect to x and y, respectively. Here, **x** is the velocity vector and **b** is a vector that contains the derivatives with respect to time.

The Least Sum of Squares (LS) is often the prime candidate for solving a set of over-determined linear equations. The main disadvantage of using LS is its sensitivity to outliers. A single outlier can produce a large error in the final results. Therefore, any LS based optic flow technique will produce erroneous results where there are motion boundaries or occluding edges.

In order to cope with motion boundaries (where there is more than one motion existing in the neighbourhood of a pixel), Fennema and Thompson (1979) proposed a clustering method using the Hough transform which is computationally very expensive. Schunck(1989) modified this method to reduce the complexity of computation by clustering the lines along the OFC produced by the central pixel. This method will produce erroneous results if the OFC of the central picture is corrupted by noise. To overcome this problem, Nesi et al.(1995) proposed a method based on the Combinatorial Hough transform, which is again computationally expensive.

The interested reader is referred to Nagle (1995) which provides a comprehensive survey on the theoretical relations between different optic flow techniques, and Barron et al. (1994) which provide a quantitative performance of many optic flow techniques.

In this paper, a new algorithm for computing the optic flow field based on robust regression techniques is presented. The major contribution of this paper is to show the relationship between the optic flow and the standard regression problem (SR). Since the SR problem is a well studied statistical problem, it will be possible to take advantage of existing robust regression techniques to find a practical solution to the optic flow problem. It will be shown in this paper that, by considering multiple motions in the neighbourhood of every pixel, and using a robust statistical method to find the dominant motion (motion of the majority of the pixels), the algorithm provides good results in the presence of noise, motion boundaries and occluding edges.

Incorporating the robustness into optic flow computation, using the least median of squares (LMS) technique (see section 3.1) is proposed here. Although the LMS method (presented in 1984 by Rousseeuw) has been popular in other areas of computer vision since early this decade (Meer et al. 1991), its importance has been over-looked by visual motion researchers (Mitiche (1994) is the only reference which mentions the possibility of using the LMS for optic flow computation, but that book does not provide any methodology). This is perhaps because Meer et al. vaguely imply that the LMS technique is not suited for the case where multiple population of data exists in a data set. To clarify this situation, we present a demanding synthetic test which clearly proves that LMS performs well even when multiple population exists. In this experiment, a set of 81 lines is considered. The narrow majority of 44 lines (53.2%) are set to go through point (2,3) while the rest of the 37 lines (46.8%) are set to go through (-1,-2). The equation **Ax**=**b** is used for creating the set of lines. Elements of matrix A are set to random numbers between [-1,1) and elements of **b** are accordingly adjusted for each line to cross the desired point. To investigate the effect of noise on the behaviour of LMS technique, normal noise is added to elements of b associated to the majority group. The results show that even if all the lines belonging to the majority are severely corrupted by normal noise and, in addition, despite the minority group being totally in agreement with the minority solution: the LMS remains faithful to the majority.

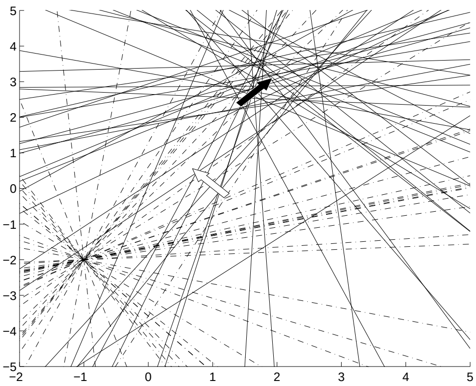

Figure 1.1 Shows an example in which the majority group are corrupted by additive normal noise with 50% amplitude. The LMS solution, black arrow pointing to (1.909,3.071), and the LS solution, white arrow pointing to (0.68610,0.55752), are also shown in this figure.

The rest of the paper is organised as follows. The next section describes the relationship between the optic flow, the closest point (CP) and the standard regression problems. A robust method for solving the optic flow problem based on the LMS technique is presented in section 3. The experimental results are presented in section 4 followed by the conclusion in section 5.

2 OPTIC FLOW AND REGRESSION

This section describes some necessary background for understanding the proposed algorithm. First the CP and SR problems are explained briefly. Then the transformation that maps one to the other will be presented. Finally, the relationship between the optic flow problem (as defined in section 1) and its associated CP problem will be described.

2.1 CP and SR Problems

To define the CP problem, it is assumed that a set of p lines defined as:

$$y = m_i x + n_i \qquad i = 1,2, ...,p \tag{1.2}$$

where m and n are constant, is given. The CP problem requires one to find the point that has the least sum of square vertical distances to this set of lines: that is, find the solution to the following minimisation problem:

$$E_{CP} = \min_{(x,y)} \sum_{i=1}^{p} (m_i x + n_i - y)^2 \tag{2.2}$$

The second problem, known as the SR problem, is to find the line ($y = mx + n$) that best fits a set of given p points (x_i,y_i). This line is the solution to the following minimisation problem:

$$E_{SR} = \min_{(m,n)} \sum_{i=1}^{p} (mx_i + n - y_i)^2 \tag{2.3}$$

The following transformation will transform the CP problem into the SR problem and vice versa:

$$\begin{cases} \mathrm{T} \quad : y = mx + n \rightarrow (-m, n) \\ \mathrm{T}^{-1}: \ (m,n) \rightarrow y = mx + n \end{cases} \tag{2.4}$$

It is easy to see that this transformation maps the E_{CP} to E_{SR} and vice versa. Having introduced this transformation, instead of solving the CP problem directly, one can transform the CP problem to a SR problem, solve the SR problem robustly and interpret the results back using the inverse transform. The main advantage of this transformation is that the SR problem is a very well studied problem and there exists a number of good robust solution methods which can now be employed to solve the CP problem.

2.2 Optic Flow and CP Problems

As shown in section 1, the optic flow problem can be simplified to the problem of solving a set of over determined linear equations. It is a well known fact that the LS solution of such a set of equations has the smallest variance among the solutions that are linear in terms of **b** (see equation 1.2) and have no systematic errors (Van Huffel and Vandewalle, 1992). This is the main reason why, in most engineering problems, the LS solution is the best one can do.

The main difference between the LS and CP approach is that in the LS approach, the equations have different contributions to the final results (based on the absolute value of their parameters); while in the CP approach all the equations have the same importance. So, the final LS solution is closer to satisfying the equations with bigger (absolute) parameters (magnitude of derivatives). In relation to optic flow, since not all parts of an image contain complete information required for motion estimation, the reliability of the different OFC, derived for different pixels, will be different. The reliability of every OFC is mainly related to the absolute values of the gradient of image brightness function (Shi and Tomasi, 1994), something which is inherently accounted for by the LS approach. Therefore, the magnitude of different residuals in the LS approach to optic flow estimation are important components of that solution, and normalising them by solving via the CP approach produces less accurate results compared to the LS approach (Bab-Hadiashar, 1996).

Although the LS problem and its associated CP problem do not have the same solution, here we accept the CP solution as an approximation for the original LS problem. The extension of this method to detect the outliers and solve the original problem using the Reweighted Least Squares is beyond the scope of this paper and will be presented elsewhere.

3 ROBUST ESTIMATION OF OPTIC FLOW

As described before, the optic flow problem can be simplified to the problem of solving an over determined system of linear simultaneous equations. This simplification is based on two assumptions: the temporal conservation of brightness and coherent motion (spatial variation of motion is smooth). Both of these assumptions are likely to be violated when there are motion boundaries, occlusion, specular reflection or transparency in the scene, or random noise in the imagery system. So, a robust solution must be able to tolerate these phenomena.

To demonstrate the effect of these on a real problem, the famous Taxi sequence is considered here. A van, in the lower right corner of this sequence, is moving toward the right with a speed of approximately 3 pixels per frame. All the derivatives of the image brightness function, in a 13 pixel wide square window centred at a point (220,130) on the van, are calculated. The linear equations relating these derivatives are plotted against the horizontal and vertical velocities (figure 3.1).

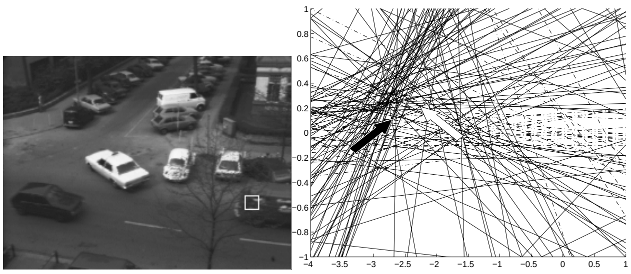

Figure (3.1): Frame 10 of the Taxi image sequence (up) and the plot of 169 lines associated with the rectangular window around the point (130,220) in the same frame. The black and white arrows point to LMS (2.862,0.209) and LS (2.262,0.211) solutions, respectively.

Careful observation of this image sequence reveals that the van is partially hidden by branches of a tree standing between camera and the van. The tree and the background of the image are stationary and therefore their associated equations (dashdot lines) are aggregated around the origin while the equations associated to the van (solid lines) are gathered around the point (-3,0). There also exists a number of other lines scattered around which are perhaps the result of some other phenomena mentioned before. This figure clearly shows the significance of the noise and other disturbing phenomena for the optic flow formulation of a real scenario.

3.1 The Least Median of Squares Technique

In this paper, the highly robust method of least median of squares (LMS) is chosen for solving the optic flow problem. In the 2-D SR problem, the LMS corresponds to finding the narrowest strip (bounded by two parallel lines) which includes one half of the data points plus one. The actual LMS line exactly halves this strip (Rousseeuw, 1984). Since the LMS solution of a linear regression problem can tolerate the outliers of up to 50 percent, the proposed method would produce sensible results as long as the majority (fifty percent plus one) of equations in the simultaneous system represent a coherent motion.

3.2 LMS Regression Line Computation

Unlike the popular LS technique, a closed form solution for computing the LMS regression line has not yet been found. The fastest known method for computing the exact LMS solution in 2-D space was introduced by Edelsbrunner and Souvaine (1990). Their algorithm involves the implementation of guided topological sweep and it requires $O(n^2)$ time and $O(n)$ space for n points in the 2-D plane.

Rousseeuw and Leroy (1987) presented an approximate algorithm for computing the LMS in an arbitrary, large dimensional, space. In this method (used in our implementation), for every set of points (equal to the dimension of space) the regression entity (ie line in 2-D or plane in 3-D, etc) is computed and the least median of square errors of all the other points in the original data set is calculated. When the smallest median of square error is computed for all the possible combinations, the set which has the smallest median of square error will be chosen as the final result. It should be noted here that this algorithm will not necessarily lead to the exact solution but it will provide a good approximation (Rousseeuw and Leroy, 1987).

4 EXPERIMENTAL RESULTS

The quantitative performance of every algorithm can be measured by applying the algorithm to a set of sequences for which the true motion fields are known. The measure of performance for every algorithm on synthetic images should be taken as an optimistic limit on the expected errors with real image sequences. This is mainly due to the fact that, in reality, the common assumptions such as uncorrelated noise, Lambertian surfaces and so on, are not always (and never 100%) met.

All the derivatives of the image brightness function for different image sequences are calculated by convolving every image sequence with a mask constructed from the derivatives of a 3-D Gaussian function with equal spatial and temporal standard deviations of 1 pixel.

The error analysis is performed using Barron's (1994) software. Therefore, the errors are reported in *degrees* and are in fact the angles between the estimated and the true motion vectors in homogeneous coordinates. As stated by Otte and Nagle (1994), the value of errors reported by this method are not uniform, and errors with equal magnitude but different directions will provide very different angular errors. Therefore, different error values reported by this method should be regarded as an indication of performance as opposed to being a strict measure of goodness.

To provide some assistance in comparing the reported results, the error statistics for the Fleet and Jepson (1990) method for every image sequence are also included. These results have been created using the publicly available software provided by Barron et al. (1994).

4.1 *New-Sinusoid1* Image Sequence

In this experiment, a sinusoidal image sequence similar to *Sinusoid1* (Barron et al. 1994) is created. The difference between this image sequence and *Sinusoid1* is that in this sequence a rectangle of 50 by 50 pixels in the middle of every images remains stationary. Therefore, there exists four linear motion boundaries inside every image.

Technique	Avg. Error	Std. Dev.	Density
Fleet and Jepson	7.39°	10.84°	43.4%
LMS (5x5)	1.42	7.30	100%

Table (4.1): *New-Sinusoid1* results on translational velocity

From this table, it can be seen that in this case, the proposed algorithm out-performs the Fleet and Jepson (1990) method both in accuracy and density.

4.2 Real Data

As mentioned before, the performance of every algorithm can only be truly investigated when it is applied to a set of real data. To indicate the performance of the proposed algorithm, the results for a real image sequence with known flow field (created by Otte and Nagle, 1994) are reported here. This image sequence is recorded using a camera which purely translates in 3-D space toward a stationary scene (with the exception of a relatively bright rectangular block shifting toward the left). Here, the results are reported for a 150 pixel-wide square window shown in figure (4.2). This window is especially challenging because it contains depth discontinuity and occlusion resulting from independent motion of the Marble block.

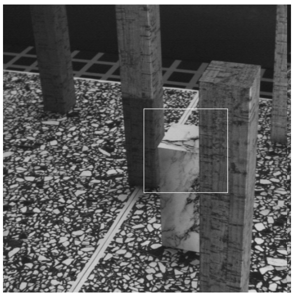

Figure (4.1): Frame No. 35 of the Otte image sequence, the white rectangle shows the window where the following results are calculated

Technique	Avg. Error	Std. Dev.	Density
Fleet and Jepson	9.56°	13.25°	44.8%
LMS (5x5)	15.73°	15.35°	100%
LMS (15x15)	11.34°	12.31°	100%
LMS (25x25)	10.53°	11.92°	100%

Table (4.2): Otte image sequence results on translational velocity frame number 35

It is shown here that although the presented method does produce results that vary with the size of window, its performance for real data is comparable to results obtained by Fleet and Jepson (1990) method. Moreover, we can prune our results to achieve a higher accuracy result for lower density (paper in preparation).

5 CONCLUSION

In this contribution, a robust method for estimating the optic flow is presented. It is shown that it is possible to detect the problematic data (containing noise, occlusion, specular reflection, etc) from good data. The presented method is highly robust and produces comparable results with the most accurate known optic flow techniques.

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